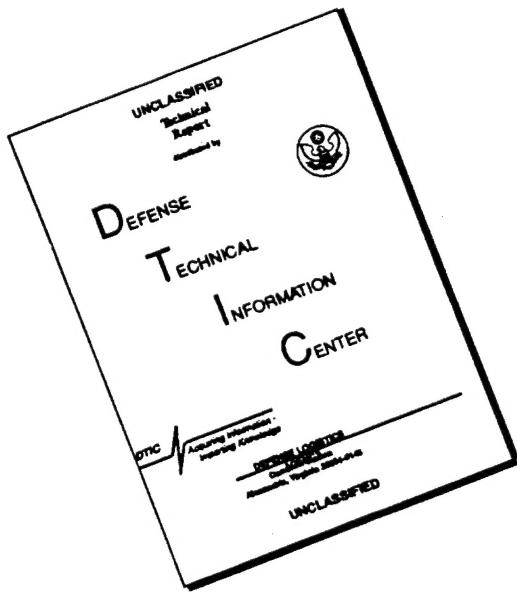


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## DESIGN AND IMPLEMENTATION OF DIGITAL CONTROLLERS ON SMART STRUCTURES USING SINGLE BOARD COMPUTERS

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Clifford Bowman,<sup>1</sup> Vittal S. Rao,<sup>1</sup> Frank J. Kern<sup>1</sup>

### ABSTRACT

The distributed control of structural systems assumes the availability of highly-integrated, embeddable control elements. Many laboratory experiments in structural control rely upon desktop computers and workstations for convenience, but the size, cost, and power requirements of general-purpose computers preclude their use in practical implementations. This study identifies and demonstrates design strategies that facilitate integration of the computational platform with other parts of an embeddable control element. Two specific strategies are explored: the first is the direct application of digital control signals to structural actuators, and the second is a constrained control-law design method that greatly reduces the necessary complexity of the digital subsystem. Each approach results in considerable simplification over the traditional digital control system and contributes to the realization of a truly embeddable element using existing technology.

### INTRODUCTION

General-purpose computers are nearly indispensable for research into structural control. Structural modeling, controller design, and system simulation are all important tasks that general-purpose computers do well, and many structural control problems would be intractable without them. General purpose computers can also find effective use in laboratory experiments involving test structures, permitting rapid prototyping and temporary implementation of control algorithms. But general purpose computers are not well suited for practical implementations: their size, cost, and power requirements render them less than ideal candidates for field use. Furthermore, the strategy of distributing control elements throughout a structure assumes compact, low-cost, embeddable control hardware -- requirements that general purpose computers cannot meet. If the ultimate goal of distributed control systems research is the "smart patch" incorporating sensor, actuator, and computational hardware into a single package, the problem of integration must itself become a subject of research.

Although advances in fabrication techniques continually extend the feasible in integrated circuits and micromachined electro-mechanical devices (O'Connor, 1992), simplification remains the most direct path to integration. The interface between structure and controller is a prime candidate for simplification, and its proper design can reduce both the computational requirements of the controller and the device-count of the interface circuitry. In terms of digital hardware, a control algorithm that limits its demands upon memory, register length, and specialized instructions can substantially reduce the complexity of the processor and its supporting circuitry. But the simplification necessary for practical implementation of a structural control system may constrain its design in ways not apparent in a well-stocked laboratory. For this reason, this study makes simplification its principal goal, seeking alternatives to the complexities of traditional laboratory design in order to achieve a hardware-limited implementation suitable for distributed control.

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Off-the-shelf single-board computers recommend themselves both as convenient first steps towards the achievement of a distributed control system and as indicators of the kinds of constraints that true distributed control will impose. The great demand for single-task microprocessor systems in industrial applications has lead to their widespread availability and has lowered their cost. Since these systems are more specialized than general-purpose computers and since they are often designed with space- and power-constraints in mind, they can offer significant practical advantages over desktop models. Readily available, compact, and power-efficient, single-board computers provide a ready solution for many practical structural-control applications. In addition, difficulties that arise from the limited precision of calculations performed on single-board computers, from their limited memories, and from their Spartan support circuitry are suggestive of the constraints likely in the design of the "smart patch."

Our principal objective is to reduce the hardware requirements of a closed-loop, structural control system. The current study employs a single-board computer to improve the response characteristics of a cantilevered beam, and although the computational power of the computer is roughly comparable to that of an IBM-PC/AT, we present a control algorithm that requires considerably less. Through appropriate sensor placement, system identification, and controller design, our implementation achieves satisfactory performance with an algorithm based solely upon elementary operations like addition, complement, and shift. In addition, we compare traditional linear actuation with direct digital excitation of the structure, presenting some PWM signal theory, an implementation example, and a performance comparison. Our results demonstrate that PWM excitation of structures achieves results comparable to those of linear actuation, and we discuss how this strategy can facilitate integration by eliminating the need for ADC's and linear amplifiers.

## SYSTEM SIMPLIFICATION OBJECTIVE

Closed-loop structural systems based upon a digital controller traditionally have the functional form shown in Figure 1. Since structural actuators generally accept continuous-time signals, the interface between these components and the digital controller usually consists of some type of digital-to-analog converter (DAC) along with any required buffering, filtering, and level-shifting circuitry. Although there are a variety of conversion schemes available, DAC's are generally highly-specialized circuits that add to the complexity of the overall system. In addition, actuation with continuous signals assumes a linear amplifier, a requirement that further complicates the design.

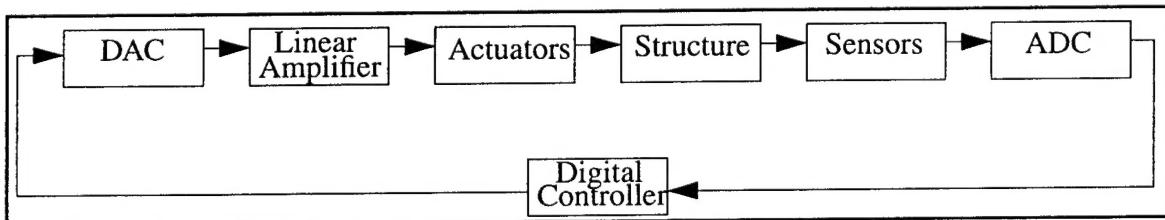


Figure 1. Block Diagram of Closed-Loop System

We propose to simplify the traditional closed-loop system by applying digital control signals directly to the actuators. This eliminates the need for the digital-to-analog converter, and we may delete its functional block entirely from the system. A second consequence of direct digital actuation is the simplification of the amplifier stage: since the amplifier output signal switches between two well-defined voltage levels, it is not necessary to compensate for crossover distortion, thermal runaway, or other sources of nonlinearity. This approach may also improve the power efficiency of the structural control system because linear amplifiers often exhibit non-negligible quiescent power dissipation. A succinct discussion of these issues may be found in (Horowitz and Hill, 1989).

## TEST ARTICLE AND COMPUTATIONAL PLATFORM

The test article for the current study consists of a cantilevered aluminum beam, bonded PZT elements for actuation and sensing, interface circuitry, and a digital subsystem. The beam is 54.0 cm long, 1.27 cm wide,

and 0.318 cm thick; two PZT elements symmetrically mounted at the beam's base provide bending-moment type actuation 4.2 cm from the clamped end, while PZT elements centered at 7.7 cm and 29.0 cm from the clamped end sense strain. Actuator placement approaches the point of maximum strain for the first mode, while the placement of the sensors results in a computationally-friendly control law as discussed below. Figure 2 summarizes the mechanical configuration of the test article.

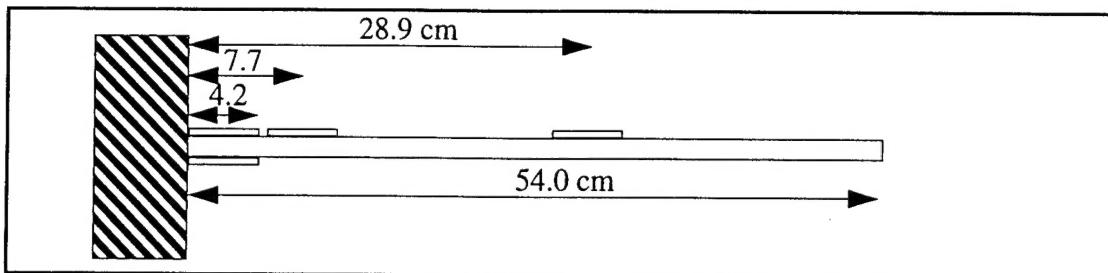


Figure 2. Simple Cantilevered Beam Test Article.

The digital subsystem is an off-the-shelf single-board computer manufactured by Intec Innoventures, Inc., of British Columbia. It is based upon Motorola's popular MC68HC16Z1 microcontroller, which includes a 16/32-bit CPU and integrated memory, timer, ADC, and PWM modules. The CPU operates at 16.78 MHz and provides instructions for 32-bit division, 16-bit multiplication, and 36-bit fixed-point multiply-and-accumulate DSP functions, although none of these capabilities are required in the current study. The standard Motorola 'Z1' configuration supports the CPU with 1024 bytes of static RAM, two 16-bit counters, eight unipolar ADC channels with selectable 8- or 10-bit resolution, two PWM output channels, a queued Serial Peripheral Interface, and numerous configurable digital ports. Intec extends the capabilities of the microcontroller by adding up to 256 Kbytes of static RAM, 256 Kbytes of Flash EPROM, a real-time clock, and buffering circuitry. The computer's peak power consumption is an estimated 1.5 watts, and the board measures 12.7 x 16.51 cm. The authors implement the controller designed below in 68HC16 assembly language, but both Motorola and Intec offer C compilers for rapid program development.

The signal conditioning circuitry that completes the system consists of a buffer and anti-aliasing filter for each input channel and level-shifting circuitry on both the input and output sides of the digital subsystem. The support circuitry included with the single-board computer provides further buffering and over-load protection for the microcontroller. To achieve linear actuation of the PZT's, the PWM output of the computer drives a low-pass filter placed before the power amplifier. With the low-pass filter removed, the control signal applied to the structure is a bi-polar pulse train that is duration-modulated according to the control algorithm.

## PULSE WIDTH MODULATED CONTROL SIGNALS

Many digital control systems devote a substantial part of their interface circuitry to the conversion of digitally-generated control signals to analog form. This is not always necessary, and in some applications digital signals can drive linear actuators directly (*e.g.* pulse-width modulated motor control). Under these circumstances, interface design becomes much simpler because there is no need for a digital-to-analog converter circuit and the power amplifier stage remains in switchmode operation. Although modeling the nonlinearities concomitant with direct digital control of actuators sometimes requires special analysis techniques (Wu and Chen, 1988), these nonlinearities are often small enough to neglect entirely. As is shown in the current example, some structural control applications are suitable for digital actuator control without any additional complexity of the model.

Pulse-width modulated (PWM) actuator control holds great promise for distributed structural control applications. Generation of PWM signals requires very little circuitry beyond a countdown timer and a clock, resources which are easily implemented with gate-array technology and may already be present in the digital subsystem for other purposes. The binary nature of the control signal keeps the succeeding amplifier stage out of the linear region and may thus improve its efficiency and simplify its design. Since most applications permit

some choice of pulse frequency, an anti-aliasing filter already present in the system can be used to prevent pulse-frequency feedthrough even with co-located sensors and actuators. As discussed above, the MC68HC16 offers two PWM outputs in its standard configuration, so currently-available microprocessors may be used for immediate implementation.

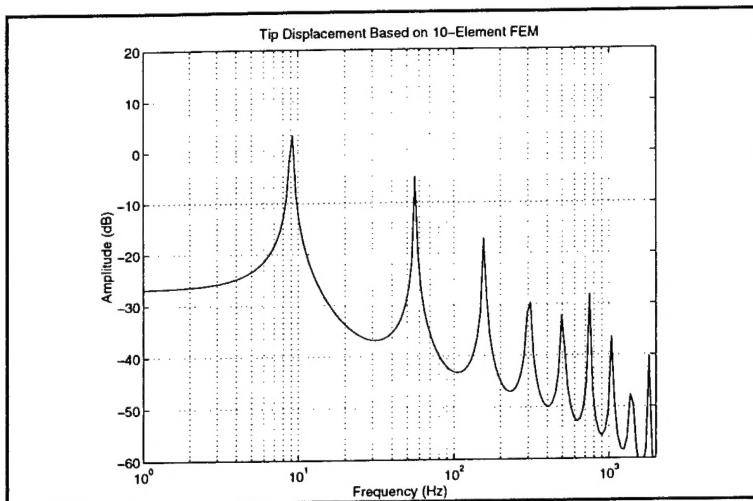


Figure 3. Tip Response to a Constant-Amplitude Excitation

PWM actuation in structural control applications is possible because the displacements due to excitation at the pulse frequency are usually negligibly small. In effect, the structure itself takes the place of a low-pass filter, responding to the modulating control signal while attenuating its carrier. For example, consider the frequency response curve shown in Figure 3 for a ten-element finite element model of the test article described above. Tip displacement tends to diminish with excitation frequency, and even when the excitation corresponds to a natural frequency of vibration the beam provides considerable attenuation at high frequencies. By choosing an appropriate pulse frequency, the system designer can often ensure that the output components due to the pulse signal are negligibly small.

The choice of pulse frequency is subject to constraints that depend upon the specific system under consideration. The digital subsystem may generate some frequencies more easily than others, and in any case timing considerations in both the digital subsystem and power amplifier will limit the highest frequency that may be used. Lower pulse frequencies may result in non-negligible excitations of the structure and may cause aliasing problems if they are not above the corner frequency of the sampling prefilter. Nevertheless, the relatively-low bandwidth of structural control systems and the continually expanding capabilities of digital hardware usually leave the designer plenty of flexibility.

The seemingly-obvious choice of a pulse frequency that does not correspond to a natural frequency of vibration requires some discussion. In the first place, the system model may either be inaccurate at high frequencies or neglect high-frequency modes altogether. Secondly, it is probably impossible to avoid exciting one or more high-frequency modes when applying a signal with as rich a spectrum as a PWM signal possesses. To see this, consider a normalized pulse train whose radian frequency is  $\omega_p$  and whose duty cycle is 50%. Expressed in series form, this signal satisfies

$$V_p(t) = \frac{1}{2} + \sum_{n=1}^{\infty} \text{sinc}\left(\frac{n \cdot \pi}{2}\right) \cdot \cos(n \cdot \omega_p \cdot t) \quad (1)$$

If the trailing edge is fully modulated by a sinusoid of frequency  $\omega_m$  the resulting signal can be expressed as

$$V_m(t) = \frac{1}{2} + \frac{1}{2} \sin(\omega_m \cdot t) + \sum_{n=1}^{\infty} \frac{2}{n \cdot \pi} \cdot \sin\left[\frac{n \cdot \pi}{2} \cdot \left(1 + \frac{1}{2} \cdot \sin(\omega_m \cdot t)\right)\right] \cdot \cos(n \cdot \omega_p \cdot t) \quad (2)$$

As can readily be seen, the high-frequency components of  $V_m(t)$  have no theoretical limit; since structural systems are themselves of infinite order, there is in general no way to guarantee that the PWM signal spectrum will not coincide with one or more of the higher-order modes.

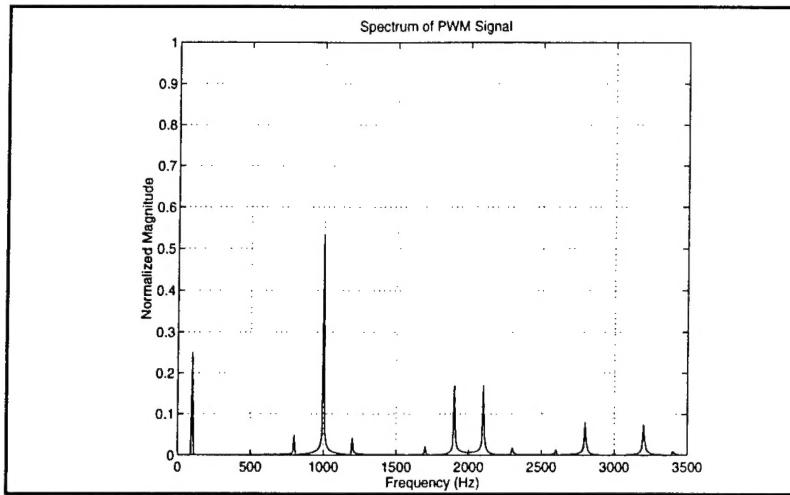


Figure 4. 1 KHz Carrier with 100 Hz Modulation

Even when a particular mode is known and modeled, it may not be possible to avoid exciting it with a PWM signal component. Consider the spectrum shown in Figure 4. In this graph, the PWM signal with 1 KHz carrier and full modulation at 100 Hz has frequency components other than those at harmonics of the carrier. In general, the frequency of these components depends upon the frequency of the modulating control signal, so predicting them at design time is difficult at best. Fortunately, it is usually sufficient to make the carrier frequency very high because even if PWM signal components excite higher-order modes, the response is usually negligible. As an illustration, Figure 5 shows a simulated “worst case” excitation of the test article, where a control signal at the beam’s fundamental frequency modulates a carrier deliberately chosen to excite the 7th mode; even under these extreme circumstances, the 35 dB difference between the system response to the modulating signal and to the carrier gives useful results. With reasonable care, the response of the system to the carrier components can be made much smaller and may be neglected in many practical applications.

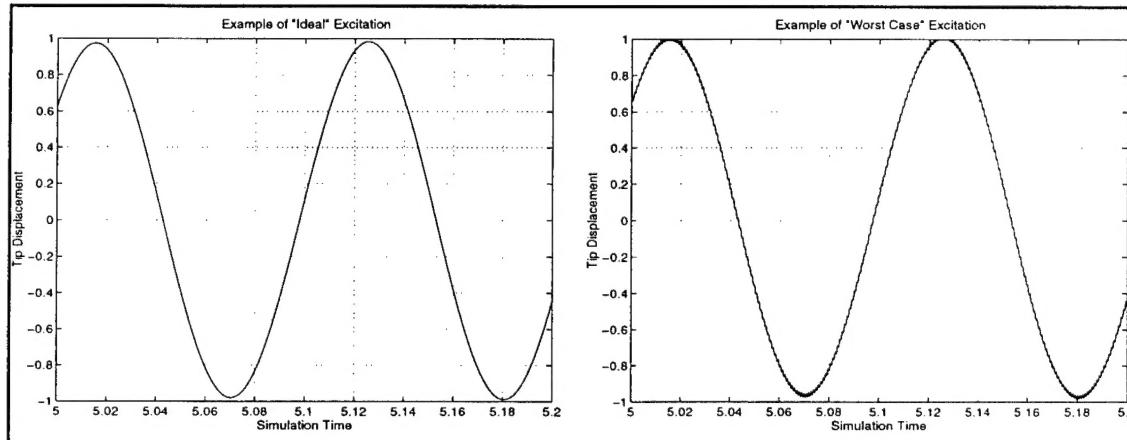


Figure 5. Example of PWM Excitation

The principal drawback of applying PWM excitations to piezoceramic actuators is the increase in the total signal amplitude. Because the signal applied to the actuators includes carrier components as well as that of the modulating signal, the overall voltage level must increase. Since the driving voltage of a piezoceramic even with linear excitation is normally quite high (Crawley and de Luis, 1990), an increase in the applied signal may sometimes be difficult to achieve. Nevertheless, in those cases where higher driving voltages are available, PWM actuation remains an attractive alternative to the linear excitation of structures.

## CONSTRAINED CONTROL-LAW DESIGN

Control algorithms that deliver adequate performance with far simpler digital platforms than are commonly used in laboratory studies are essential to the development of truly embeddable systems. The authors believe that the ideal platform from the standpoint of system integration would be based upon non-proprietary and power-efficient technologies like gate arrays. Since such implementations are contingent upon limiting the computational demands of the control algorithm, research into new design methods is of critical importance. Previous studies have established methods for identifying multiple-input multiple-output (MIMO) system models suitable for full state feedback control with simple analog hardware (Butler and Rao, 1995). These methods are of particular interest because they eliminate the need for computationally-demanding state estimation. Although manufacturing repeatability and application-specificity limit the attractiveness of analog control in an integrated application like the smart patch, an appropriate combination of gate-implementable operations like addition, negation, and arithmetic shifting can approximate the analog functions of differentiation, summation, and multiplication. Application of these methods along with a constrained control-law design strategy can result in a computationally-friendly algorithm that requires little processor complexity.

Let us consider the state-variable representation of a general  $n^{\text{th}}$  order system.

$$\begin{aligned}\dot{x} &= A_{OL}x + B\dot{u} \\ \dot{y} &= Cx\end{aligned}\tag{3}$$

The desired closed-loop eigenvectors and eigenvalues will in general differ from those of the open-loop system, and we designate the  $i^{\text{th}}$  desired eigenvalue by  $\lambda_i$  and the  $i^{\text{th}}$  desired eigenvector by  $\psi_i$ . The closed-loop system must satisfy  $n$  equations

$$\begin{aligned}(\lambda_1 I - A_{OL})\psi_1 + BK\psi_1 &= 0 \\ (\lambda_2 I - A_{OL})\psi_2 + BK\psi_2 &= 0 \\ &\vdots \\ (\lambda_n I - A_{OL})\psi_n + BK\psi_n &= 0\end{aligned}\tag{4}$$

Now let

$$T = [\psi_1 \ \psi_2 \ \dots \ \psi_n]\tag{5}$$

and

$$\Lambda = \begin{bmatrix} \lambda_1 & 0 & \dots & 0 \\ 0 & \lambda_2 & \dots & 0 \\ \vdots & \vdots & \vdots & \vdots \\ 0 & 0 & \dots & \lambda_n \end{bmatrix} \quad (6)$$

Then we may rewrite Equation (3)

$$BKT = A_{OL}T - T\Lambda \quad (7)$$

or

$$BK = A_{OL} - T\Lambda T^{-1} = (A_{OL} - A_{CL}) \quad (8)$$

In traditional controller design, we specify the closed-loop response (and hence,  $A_{CL}$ ), accept  $A_{OL}$  and  $B$  as fixed, and find a suitable gain-law  $K$ . In constrained control-law design, we wish to specify the form of  $K$  and  $A_{CL}$ , so the input weighting matrix  $B$  is what we must find. The general constrained control-law design problem thus hinges on suitable actuator placement and control-signal conditioning.

Under certain circumstances, we may manipulate Equation (8) to more convenient forms. If it is necessary to control all  $n$  modes independently, we set  $m=n$  and choose a computationally-friendly  $K$  with full rank. This leads to

$$B = (A_{OL} - A_{CL})K^{-1} \quad (9)$$

Alternately, we may allow  $m < n$  and choose  $(n-m)$  modes whose eigenvectors and eigenvalues will not be changed under closed-loop control. For these modes

$$BK\psi_i = 0 \quad (10)$$

which constrains

$$K' \perp \psi_i \quad (11)$$

Thus the feed-back control law  $K$  is constrained so that its rowspace is orthogonal to the  $(n-m)$  unchanged eigenvectors. This suggests a strategy whereby we may constrain  $K$  to some desired form by “shaping” the unchanged  $\psi_i$ 's. The implementation below is an example of this approach with  $m=1$ .

Consider a single-output system like the one described above. Our constrained control-law strategy for this system arises from the observation that if an eigenvector of the open-loop system matrix  $A_{OL}$  remains unchanged by the operation

$$A_{CL} = A_{OL} - BK \quad (12)$$

(and thus is an eigenvector of the closed-loop system matrix,  $A_{CL}$ ) then it must lie in the nullspace of the product  $BK$ . If  $B$  and  $K$  are both nonzero, then  $BK$  will be nonzero with rank 1, and the unchanged eigenvector must be orthogonal to  $K^T$ . To put this observation to constructive use, the designer uses sensor placement or signal conditioning to shape the eigenvector of the higher-order mode to be orthogonal to the desired  $K$ ; then by leaving the

higher-order mode unchanged,  $K$  is constrained to be orthogonal to the higher-order eigenvector. By these means, the designer determines the shape of the state-feedback gain matrix and can choose computationally-friendly “shapes” for it to assume. Note that in the case of multiple inputs, these results remain valid provided that actuator placement results in linear independence among the columns of  $B$ .

## DESIGN AND IMPLEMENTATION EXAMPLE

In order to apply the constrained control law strategy described above, the goal of a computationally-friendly feedback matrix must drive each step of the design. Even before the sensor elements are applied to the beam, we must analyze its mechanical behavior in order to achieve an appropriate form for the second-mode eigenvector of the system. Choosing the desired  $K$  to be of the form

$$K = \begin{bmatrix} 2k_1 & k_1 & -2k_2 & -k_2 \end{bmatrix} \quad (13)$$

we see that we require a second-mode eigenvector of the form

$$\Psi_2 = \begin{bmatrix} -\Psi_{21} & 2\Psi_{21} & -\Psi_{22} & 2\Psi_{22} \end{bmatrix}^T \quad (14)$$

in order to achieve the required orthogonality. Consequently, we refer to the strain function for a cantilevered beam and find that the locations  $x_1 = 7.7$  cm and  $x_2 = 28.9$  cm will experience strains in the appropriate proportions to produce the first two states. Since the last two states are simply time derivatives, we see that their ratio will follow that of the first two states. Based on these calculations, we place the sensors and begin the modeling process.

Our next step is to measure the eigenvalues and eigenvectors of the open-loop system. Applying the elliptical test for natural frequency measurements (James *et al.*, 1989), we find that the first two modes of vibration for the test structure correspond to excitations at 9.53 Hz and 57.51 Hz. By measuring the sensor outputs at each of these frequencies and calculating their time derivatives, we estimate the transformation and undamped diagonalized eigenvalue matrices to be

$$T = \begin{bmatrix} 5 & 5 & -2.75 & -2.75 \\ 2.3 & 2.3 & 5.5 & 5.5 \\ 1.682j & -1.682j & -5.58j & 5.58j \\ 0.774j & -0.774j & 11.2j & -11.2j \end{bmatrix} \quad (15)$$

and

$$\Lambda_u = \begin{bmatrix} j59.9 & 0 & 0 & 0 \\ 0 & -j59.9 & 0 & 0 \\ 0 & 0 & j361.4 & 0 \\ 0 & 0 & 0 & -j361.4 \end{bmatrix} \quad (16)$$

Measuring the damping for each mode to be 0.5%, we next write the damped diagonalized eigenvalue matrix,  $\Lambda_d$ ,

$$\Lambda_d = \begin{bmatrix} -0.3 + j59.9 & 0 & 0 & 0 \\ 0 & -0.3 - j59.9 & 0 & 0 \\ 0 & 0 & -1.8 + j361.3 & 0 \\ 0 & 0 & 0 & -1.8 - j361.3 \end{bmatrix} \quad (17)$$

and calculate the system matrices:

$$A = T\Lambda_d T^{-1} = \begin{bmatrix} -0.581 & 0.613 & 178 & -0.0341 \\ 0.564 & -1.52 & 0.183 & 178 \\ -153 & 290 & -0.581 & 0.611 \\ 268 & -602 & 0.564 & -1.53 \end{bmatrix} \quad (18)$$

and

$$B = T \sum_{i=1}^4 k_i [j\omega_i I - \Lambda_d] T^{-1} (j\psi_i) = \begin{bmatrix} 0 \\ 0 \\ 19.16 \\ -40.93 \end{bmatrix} \quad (19)$$

where  $\omega_2 = -\omega_1$ ,  $\omega_4 = -\omega_3$ , and  $k_i$  represents a gain constant associated with the sampling rate.

Our control objective is to increase the damping of the first mode. A convenient value for closed-loop damping is 1.5%, for which we calculate the gain law

$$K = [\overrightarrow{\alpha - a}] \mathcal{A}^{-1} \tilde{C}^{-1} = [0.0033 \ 0.0016 \ -1.934 \ -0.968] \quad (20)$$

We verify that the gain law has the desired shape and that the second mode eigenvector lies in the nullspace of  $BK$ . A further simplification results when we approximate this gain law by

$$\hat{K} = [0 \ 0 \ -2 \ -1] \quad (21)$$

which meets our requirements for computational simplicity.

We implement this control algorithm using the single-board computer described above. Although this board has a minimum sampling rate of 11 KHz, through decimation it is possible to sample the pre-filtered sensor outputs at a nominal rate of  $f_s = 712$  Hz. We then use the function

$$\frac{\Delta y}{\Delta t} = f_s \times \frac{y(k) - y(k-2) + y(k-1) - y(k-3)}{4} \quad (22)$$

to approximate the velocity at each sensor location, and we retain every fourth derivative calculation and every fourth displacement measurement to achieve an effective sampling rate of 178 Hz. Although this is not multi-rate sampling in the usual sense, it is an effective strategy that improves the accuracy of the digital differentiation while avoiding the difficulties of oversampling. Note that this calculation reduces to addition, negation, and shifting operations when  $f_s$  term is subsumed by the  $k_i$  terms of Equation (19).

Once the derivatives of the sensor outputs are calculated, it is a simple matter to implement the gain law. Using the approximation in Equation (21), two registers suffice for the calculation of the control signal, and since the controller design assumed negative feedback, no negation operation is required. With linear actuation, the closed-loop system achieves 2.67% damping of the first mode, slightly more than the design value. The closed-loop system exhibits 2.53% damping when driven directly with PWM control signals at the same effective volt-

age levels. In each case, the observed damping is slightly greater than that predicted by the model; these discrepancies are attributable to the approximation used in Equation (21) and to modeling error. Figure 6 summarizes these results.

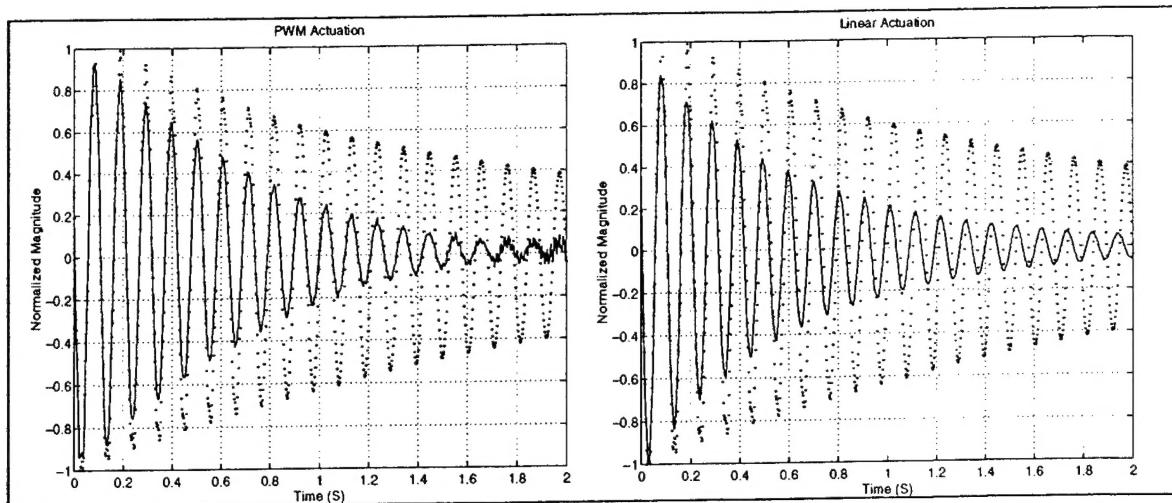


Figure 6. Experimental Results

## CONCLUSIONS

This study demonstrates a structural control implementation strategy that limits the hardware requirements of the closed-loop system. Gain law shaping based upon sensor placement results in a feedback control algorithm that can be implemented with addition, negation, and shifting operations; consequently, this procedure significantly reduces the required complexity of the digital subsystem. Direct digital excitation of a structural system is demonstrated, resulting in simplification of the controller/structure interface and achieving satisfactory results. The study discusses the limitations of both aspects of this implementation.

Since the control problem chosen for this study involves the control only of a single mode, constrained design for multi-mode systems is a prime candidate for future research. The particular characteristics of the distributed control problem -- such as the abundance of sensors and flexibility in their placement, analytic knowledge of the structure's behavior, and the possibility of modal isolation -- might be exploited in other ways to achieve a computationally-desirable design. The authors plan to investigate gain law shaping strategies that involve distributed sensors in the near future.

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